ANALYTICAL DETERMINATION OF THE EQUILIBRIUM TEMPERATURE OF AN ADIABATICALLY EVAPORATING LIQUID

V. A. Maiorov UDC 536.423.1

A simple analytical expression is derived to determine the equilibrium temperature of an adiabatically evaporating liquid (true temperature of a wet-bulb thermometer). Comparison of the calculated data with well-known experimental results confirms the rather high degree of their coincidence in the entire investigated range of parameters of the process of water evaporation into a humid air flow.

In the processes of heat and mass exchange in evaporation of a liquid from a free surface into a vapor-gas medium, of special value is the magnitude of the liquid temperature in the case of equilibrium adiabatic evaporation, when the entire convective heat flux from a vapor-gas mixture is spent to evaporate the liquid. This temperature is called the true wet-bulb thermometer temperature t_w. The temperature of equilibrium evaporation is that characteristic quantity to which the temperature of the liquid surface is related in some way: depending on the conditions of heat supply to the liquid, the temperature of its surface is either equal to the equilibrium one, or closely coincides with it, or is determined in terms of it [1]. For a humid air the true wet-bulb thermometer temperature is one of the main parameters of its state. However, up to now there has been no analytical dependence that would make it possible to rather easily and exactly calculate this most important characteristic of the process.

Let us consider evaporation of liquid from a free surface into a vapor-gas mixture flow whose temperature t_0 (dry thermometer temperature) is higher than the liquid surface temperature t_s. In this case, to determine the density of a convective heat flux from the vapor-gas mixture to the liquid surface

$$q = \alpha \, (t_0 - t_s) \tag{1}$$

it is necessary to know its temperature t_s, whereas to determine the density of the mass flux of the evaporated liquid

$$j = \beta \, (C_s - C_0) \tag{2}$$

it is necessary to know the concentration of the liquid vapors near the surface C_s, with the quantities C_s and t_s being interconnected by the dependence of vapor density on temperature in the state of saturation $C_s = C_{sat}(t_s)$. The concentration of the liquid vapor in the external flow C_0 is known.

In adiabatic evaporation, when the whole heat supplied is spent to evaporate the liquid

$$q = rj \,, \tag{3}$$

the temperature of its surface is equal to the temperature of the whole liquid and is equal to the true wet-bulb thermometer temperature $t_s = t_w$:

1131

$$\alpha\,(t_0 - t_w) = r\beta\,(C_w - C_0)\,. \tag{4}$$

Whence an expression is obtained to calculate the true wet-bulb thermometer temperature t_w or the difference of the temperatures of the external flow and liquid $t_0 - t_w$ (which in processes with a humid air is called the psychrometric difference):

$$t_0 - t_w = (r\beta/\alpha)\,[C_w\,(t_w) - C_0]\,. \tag{5}$$

The concentration of the liquid vapor near the surface is uniquely determined by its temperature t_w with the aid of the dependence of vapor density on temperature in the state of saturation:

$$C_w = C_{sat}\,(t_w)\,. \tag{6}$$

Proceeding from conditions (5) and (6), we derive an analytic expression for calculating the psychrometric difference $t_0 - t_w$. Two assumptions are used for this purpose.

The first assumption is that the processes of heat- and mass exchange are similar. In this case, in the dimensionless equation of heat exchange (for example, for forced convection)

$$\mathrm{Nu} = C\,\mathrm{Re}^n\,\mathrm{Pr}^m\,, \tag{7}$$

$$\mathrm{Nu} = \alpha l/\lambda\,, \quad \mathrm{Re} = ul/\nu\,, \quad \mathrm{Pr} = \nu/a \tag{8}$$

and in the dimensionless equation of mass exchange

$$\mathrm{Sh} = C\,\mathrm{Re}^n\,\mathrm{Sc}^m\,, \tag{9}$$

$$\mathrm{Sh} = \beta l/D\,, \quad \mathrm{Sc} = \nu/D \tag{10}$$

the values of the Re numbers, coefficients c, n, and m, and characteristic dimension l are identical. Taking this into account and dividing term by term the left- and right-hand sides of Eqs. (9) and (7), we obtain a simple relationship between the coefficients of mass exchange and heat exchange:

$$\beta/\alpha = (D/\lambda)\,(\mathrm{Sc}/\mathrm{Pr})^m\,. \tag{11}$$

The second assumption is that the liquid vapor is an ideal gas:

$$C = (P\mu)/(R_\mu T)\,, \tag{12}$$

and that the saturation curve for the liquid is described by the Clapeyron equation [2]

$$P_1/P_0 = \exp\,[(r\mu/R_\mu)\,(1/T_0 - 1/T_1)]\,. \tag{13}$$

Here the subscript 1 denotes the state of saturation at the absolute temperature of the evaporating liquid $T_1 = t_w + 273.15$ and the subscript 0 denotes the state of saturation at the absolute temperature of the external flow $T_0 = t_0 + 273.15$.

Substituting Eqs. (11)–(13) into Eq. (5), we obtain the relation

$$t_0 - t_w = (Dr/\lambda)\,(\mathrm{Sc}/\mathrm{Pr})^m\,(P_0\mu/R_\mu T_0)\,[(T_0/T_1)\,(P_1/P_0) - \varphi_0]\,, \tag{14}$$
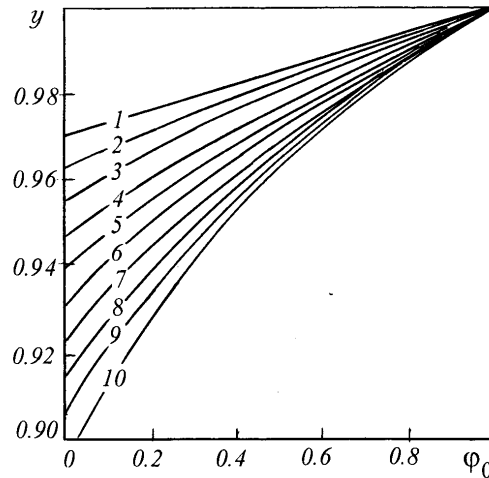
1132

Fig. 1. The solution $y = y(a, b, \varphi_0)$ of the characteristic equation (16) for the process of water evaporation into a flow of humid air with a different temperature according to the dry thermometer t_0: 1) $t_0 = 5°C$; 2) 10; 3) 15; 4) 20; 5) 25; 6) 30; 7) 35; 8) 40; 9) 45; 10) 50.

which is reduced to the form

$$(t_0 - t_w)/T_0 = (D/\lambda)\,(Sc/Pr)^m\,(P_0/T_0)\,(r\mu/R_\mu T_0)\,\left\{(T_0/T_1)\exp\left[(r\mu/R_\mu T_0)\,(1 - T_0/T_1)\right] - \varphi_0\right\}, \qquad (15)$$

and finally we write in dimensionless form

$$\exp\left\{-a\,[(1-y)/y]\right\}/y - (1-y)/ab = \varphi_0. \qquad (16)$$

The latter equation contains the dimensionless parameters

$$y = T_1/T_0, \quad a = r\mu/R_\mu T_0, \quad b = (D/\lambda)\,(Sc/Pr)^m\,(P_0/T_0), \quad \varphi_0 = C_0/C_i\,(t_0), \qquad (17)$$

where φ_0, equal to the concentration ratio of vapor in the free stream and of the saturated vapor at its temperature t_0, denotes relative humidity for a humid air; P_0 is the pressure of the saturated vapor of the liquid at t_0.

The dimensionless parameter a depends only on the physical properties of the liquid vapor r and μ and of the external flow temperature t_0. The parameter b is determined by the physical properties of the liquid vapor D and P_0 and of the external gas flow λ and t_0. The conditions of the convective heat and mass exchange determined by relations (7)–(11) are represented by the ratio $(Sc/Pr)^m$ in the parameter b.

Analyzing Eq. (16), one can note the following. If we assume that, other conditions being constant (constant a and b), the concentration of the liquid vapor in the main stream φ_0 may be prescribed independently, then Eq. (16) can be considered as the characteristic equation whose numerical solution allows one to find the dependence of the equilibrium temperature of the liquid on the vapor concentration in the external flow, i.e., $y = y(a, b, \varphi_0)$. If in any way one manages to measure the temperature of the surface of the liquid, then Eq. (16) can be considered as an analytical expression that allows one to easily determine the concentration of vapor in the flow (relative humidity) as a function of t_w, i.e., $\varphi_0 = \varphi_0(a, b, y)$.

We compare the obtained analytical results (16) and (17) with experimental data. The largest number of these data were obtained for the process of water evaporation into a humid air, for example, when the relative humidity of air was measured with the aid of a psychrometer using the reading of the dry t_0 and wet-bulb t_w thermometers.
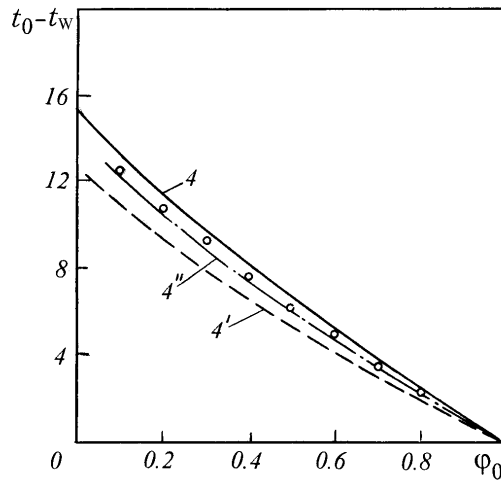
1133

Fig. 2. Comparison of the calculated true (4) and calculated-experimental psychrometric differences for a humid air with the temperature according to the dry thermometer $t_0 = 20°C$: 4′) the calculated-experimental difference for air flowing with a velocity of 0.8 m/sec near the wetted thermometer; 4″) the same for a velocity of 2 m/sec; points, the data determined from the psychrometric diagram of [1] and from the i–d diagram of a humid air.

The form of the dependence $y = y(a, b, \varphi_0)$ determined by expressions (16) and (17) for the case of water evaporation in a flow of a humid air at atmospheric pressure is presented in Fig. 1. The external flow temperature t_0 was taken as the basic parameter. The properties of a dry air were taken as the physical properties of the external flow, namely, at the selected values of t_0 the saturated steam pressure is small in comparison with the atmospheric one and the concentration of water vapor in the external flow is insignificant even when $\varphi_0 = 1$, and they do not influence the physical properties of the external flow. (In evaporation of all other liquids into air, there are usually no vapors of these liquids in the external flow $\varphi = 0$ and they do not influence the physical properties of the external flow.) When t_0 changes from 5 to 50°C, the parameter a decreases from 19.39 to 15.97 and the parameter b increases from 0.00276 to 0.03863. The diffusion coefficient for the air–steam mixture was calculated by the following relation from [3]: $D = D_0(T_0/273)^{1.81}$, $D_0 = 2.23 \cdot 10^{-5}$ m²/sec. In calculating the parameters b, the value of the exponent m in $(Sc/Pr)^m$ was taken to be equal to $m = 1/3$; the value of $(Sc/Pr)^{1/3}$ differs little from unity and remains practically constant 0.945–0.947.

Figure 2 shows (solid curve 4) the dependence (calculated with the aid of Eq. (16)) of the true psychrometric difference $t_0 - t_w$ on the relative moisture content φ_0 of the external flow of a humid air having a temperature of $t_0 = 20°C$. This curve is obtained from the corresponding curve 4 in Fig. 1 by means of the conversion $t_0 - t_w = (1 - y)(t_0 + 273.15)$.

For comparison, the dashed line 4′ depicts the semianalytical dependence obtained in determining the relative humidity of the atmospheric air by a stationary psychrometer, used at meteorological stations, at a ventilation velocity of a wet thermometer of 0.8 m/sec. These data are cited in official psychrometric tables [4].

The comparison of the presented calculated (solid curve 4) and semianalytical (dashed curve 4′) results shows that the analytical true psychrometric difference $(t_0 - t_w)$ exceeds the experimental difference $(t_0 - t_w')$ in the entire range of change of the relative air humidity, with the difference between these curves remaining practically constant in the entire range of φ_0 and coming to 17–18%. The main reason is that the wetted thermometer shows the temperature t_w' to be always higher than the true wet-bulb temperature t_w, with
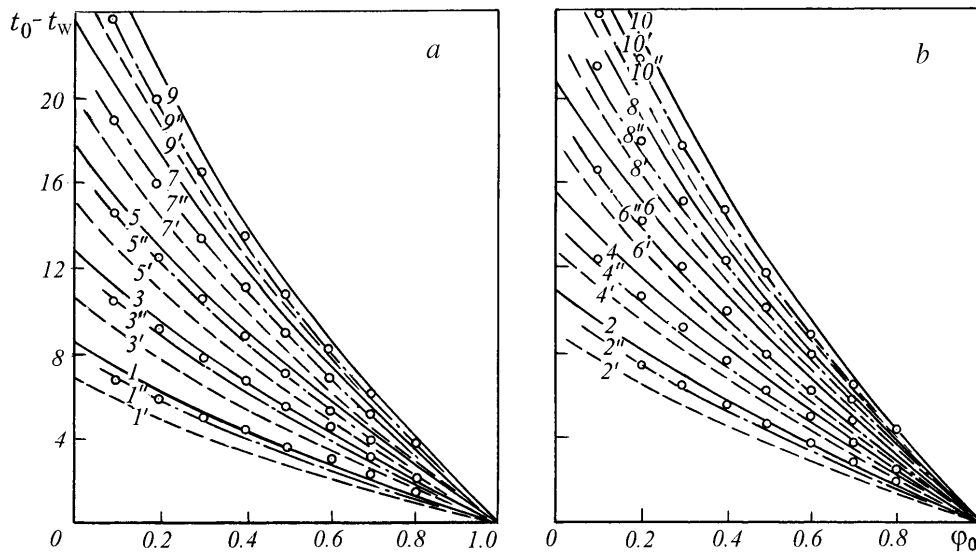
Fig. 3. Calculated and calculated-experimental psychrometric differences for a humid air at different temperatures of air according to the dry thermometer t_0: (a) $t_0 = 5^{\circ}$C; 3) 15; 5) 25; 7) 35; 9) 45; (b) 2) $t_0 = 10^{\circ}$C; 4) 20; 6) 30; 8) 40; 10) 50.

the difference between the measured temperature t'_w and the true one t_w decreasing with increase in the air-flow velocity near the wetted thermometer.

The psychometric tables of [4] contain correction data that make it possible to make corrections and use the main body of the data of these tables for determining the humidity of air with the aid of an aspiration psychrometer at an air-flow velocity of 2 m/sec near the wetted thermometer. The dashed-dotted curve $4''$ (see Fig. 2) presents the psychrometric difference $(t_0 - t''_w)$, measured by such a tool, as a function of the relative air humidity. An increase in the air velocity near the wetted thermometer from 0.8 to 2.0 m/sec leads to a noticeable increase in the measured psychrometric difference and to a decrease of its difference from the true value of $(t_0 - t_w)$ — the difference amounts to 17–18% for 4 and $4'$ and decreases to 7–8% for curves 4 and $4''$, with the difference between the measured data $4'$ and $4''$ being equal to 10%. The true curve 4 is the limit for the computational-experimental curves of the type $4'$ and $4''$ with a further increase in the air velocity near the wetted thermometer.

The points in Fig. 2 denote the values of the psychrometric difference determined by means of the psychrometric diagram of [1] and the $i–d$ diagram of humid air constructed for atmospheric pressure. These results can be considered as standard. They virtually coincide with the dash-dotted curve $4''$.

The analytical and calculated-experimental results for the variant of water evaporation into a flux of humid air with a dry thermometer temperature of $t_0 = 20^{\circ}$C are represented in Fig. 2. Similar results for the temperatures of humid air according to a dry thermometer in the range from 5 to 50°C with an interval of 5°C are depicted in Fig. 3. They are given in two figures to avoid overlapping of the data. The system of designations is the same as in Fig. 2.

The results for all the temperatures are similar to those given in Fig. 2 for the temperature $t_0 = 20^{\circ}$C; for ease of comparison they are repeatedly depicted in Fig. 3a. For humid air fluxes with a temperature above 40°C, there is a gradual deviation of the data determined by the diagrams (light points) from the dash-point curves in the direction of calculated curves.

The deviation of the semianalytical difference $(t_0 - t'_w)$ from the predicted one $(t_0 - t_w)$ for a stationary psychrometer for air moving with a velocity of 0.8 m/sec decreases monotonically from 20 to 10% with increase in the air temperature, according to the dry thermometer, from 5 to 50°C. For an aspiration psy-
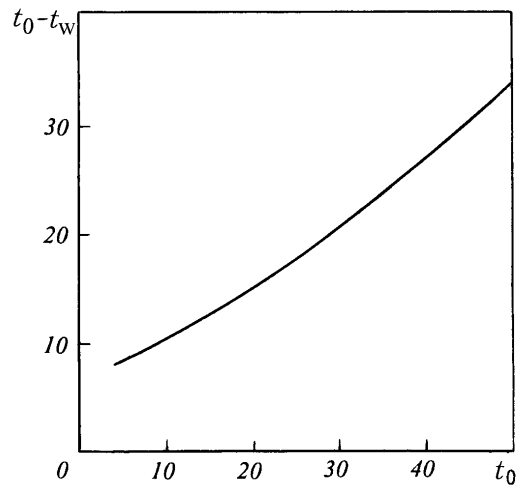
1135

Fig. 4. Dependence of the maximum true psychrometric difference on the temperature of a dry ($\varphi_0 = 0$) air. t, $^\circ$C.

chrometer with an air ventilation rate of 2 m/sec, the deviation of the psychrometer difference ($t_0 - t_w''$) from the true calculated one ($t_0 - t_w$) is much smaller (not higher than 6–10%).

Figure 4 presents the dependence of the maximum psychrometric difference on the temperature of the air flow attained in the limiting regime of evaporation of water into the dry air flow ($\varphi_0 = 0$). The semianalytical data for these conditions are absent in the psychrometric tables of [4].

The results presented allow the conclusion that the values calculated according to Eq. (16) represent the true wet-bulb thermometer temperature or differ from it by no more than 6–8% in the entire investigated range of parameters. Therefore, this model can be considered as a rather accurate first approximation to the truth, at least, in the range of comparison with experimental results, i.e., with the difference between the temperatures of the dry and wet-bulb thermometers up to 25°C.

NOTATION

C, concentration of vapors; D, coefficient of diffusion; j, mass-flux density; P, pressure; q, heat-flux density; r, specific heat of vapor generation; R_μ, universal gas constant; t, temperature (practical scale); T, absolute temperature; u, external flow velocity; α, heat-transfer coefficient; β, mass-transfer coefficient; λ, thermal conductivity; μ, molecular constant of vapors; ν, kinematic coefficient of viscosity; φ, relative concentration of liquid vapors in a flow. Similarity numbers: Nu, Nusselt number; Re, Reynolds number; Pr, Prandtl number; Sh, Sherwood number; Sc, Schmidt number. Subscripts: 0, external flow; w, the surface of adiabatically evaporating liquid (wet-bulb thermometer); sat, saturation; s, surface.

REFERENCES

1. A. V. Nesterenko, *Principles of Thermodynamic Calculations of Ventilation and Conditioning of Air* [in Russian], Moscow (1971).
2. R. C. Reid, J. M. Prausnitz, and T. K. Sherwood, *The Properties of Gases and Liquids* [Russian translation], Leningrad (1982).
3. N. B. Vargaftik, *Handbook on Thermophysical Properties of Gases and Liquids* [in Russian], Moscow (1972).
4. D. P. Bespalov, L. T. Matveev, V. N. Kozlov, and L. I. Naumova, *Psychrometric Tables* [in Russian], Leningrad (1981).